Lattice Study of the H Dibaryon

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The mass of the lowest spin-zero, strangeness-(-2) flavor singlet state in the dibaryon sector has been calculated in quenched QCD on $16^3 \times 32$ and $24^3 \times 32$ lattices at $\beta = 5.85$ to study whether the energy of the proposed H dibaryon is near or below the $\Lambda\Lambda$ threshold. Preliminary results indicate that finite lattice volume artifacts overestimate the binding, and that on the largest lattice m_H is of the order of 100 MeV above the $\Lambda\Lambda$ threshold.

1. Introduction

Based on physical bag model arguments and the magnetic hyperfine interaction, Jaffe suggested that the lowest bound state in the dibaryon sector would be a spin 0 strangeness -2 $SU(3)$ flavor singlet, and that the energy of this H dibaryon could be near or below the $\Lambda\Lambda$ threshold [1]. Despite two decades of effort, experiments have neither found it nor excluded the possibility of exotic multiquark systems ($Q^n\bar{Q}^m$, for $n+m > 3$).

If a six quark system exists as a bound state, a single gluon exchange will contribute

$$\Delta = -\sum_{ij}(\vec{\sigma}_i \cdot \vec{\sigma}_j)(\vec{\lambda}_i \cdot \vec{\lambda}_j)M_{ij} \tag{1}$$

to the mass [1] yielding Jaffe's original bag model estimate of $m_H = 2150\,\text{MeV}$, 81 MeV below the 2231 MeV $\Lambda\Lambda$ threshold. Subsequent bag model predictions [2] yield the mass range $1.03 -$ 2.3 GeV. Although early lattice calculations discussed below were not definitive [3,4], the success of contemporary spectroscopy in quenched QCD motivated us to attempt a new lattice calculation of the H dibaryon.

2. Lattice Parameters

We have studied two lattice sizes, $16^3 \times 32$ and $24^3 \times 32$ at quenched $\beta = 5.85$, corresponding to a lattice spacing $a \approx 0.13(3)$ fm. Wilson fermion propagators were calculated at seven values of the hopping parameter for the bare quark mass in the range $30 - 300\,\text{MeV}$ using a point source at $t = 0$. For the fermions the boundary conditions were periodic in spatial directions and hard wall (Neumann) in the temporal direction. This choice allowed us to obtain a signal at large euclidian times which is crucial for the calculation. A total of 56 configurations at $16^3 \times 32$ and 40 at $24^3 \times 32$ were used.

Once the quark propagators were calculated, the two-point functions for the H as well as for assorted mesons (π, ρ, ϕ, K and K^*) and hadrons (N, Λ and Ξ) were constructed with the three-momentum $\vec{p} = 0$ at the sink. The lattice spacing is defined by n_N, the degenerate u and d quark masses are defined by m_π/m_N and the strange quark mass is defined by m_Λ/m_N. These definition were chosen to absorb as many of the errors arising from the quenched approximation as possible.

3. H Two-point Function

While the H propagator could be expressed in terms of two-hadron states, this form is not computationally feasible due to numerous duplicate terms and mutual cancellations. Instead, we constructed the polynomial arising from contracting $J = 0$, $S = -2$, $B = 2$ sources and sinks of the form $\phi_H = \sum c_i \epsilon^{abc}\epsilon^{a'b'c'}\epsilon_{\alpha\beta}\epsilon_{\gamma\mu}\epsilon_{\nu\lambda}(uuddss)^{\alpha\beta\gamma\mu\nu\lambda}_{abca'b'c'}$, and manipulated it analytically to reduce the computational complexity by a factor of 200. The final expression was calculated by two independent meth-

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ods and symmetries of the result were checked.

As a result, the machine time was divided evenly between the Dirac inverter and two-point function calculations for 16 combinations of heavy and light quark masses.

4. Measurements

In the absence of finite boundary effects, one may identify the lowest mass by writing the zero momentum two point function

$$D(t) = \int d^3x\, e^{ipx} D(0;x,t)|_{p=0} = \sum_{|n\rangle} a_n e^{-\langle n|\hat{H}|n\rangle t} \qquad (2)$$

where n runs over all states with given quantum numbers and extracting the exponent with smallest mass. In our case, there is an additional correction arising from the hard wall, so we approximate $D(t)$ in the region of interest by three terms:

$$D(t) \approx a_1 e^{-m_1 t} + a_2 e^{-m_2 t} + a_3 e^{-m_3 t}, \qquad (3)$$

and fit data in the window $t_1 \le t \le t_2$. In this case, m_1 represents contributions from the excited states, m_2 is the ground state mass and m_3 accounts for the wall effects. This fit has been chosen over a simpler one mass fit because it is less sensitive to the window size and position. In addition, the resulting values of m_2 are less contaminated by higher excitations and wall effects. Typical results of the fit to the Λ and H on the $24^3 \times 32$ lattice are shown on figures 1 and 2 respectively. As one can see, the wall effects on baryons are rather small, while, as expected from the much more rapid fall off for the dibaryon, there is a larger effect on the H. We note that in all cases, eq. (3) yielded a good fit to the linear interior region that was stable against reasonable changes in the (t_1, t_2) window. Errors in the mass m_2 were determined by least squares analysis. Physical masses were determined by linear extrapolation in the light and strange quark masses.

5. Baryon Spectrum

Since m_u and m_d are fixed by the nucleon mass, in the non-strange sector, m_ρ is a test both for

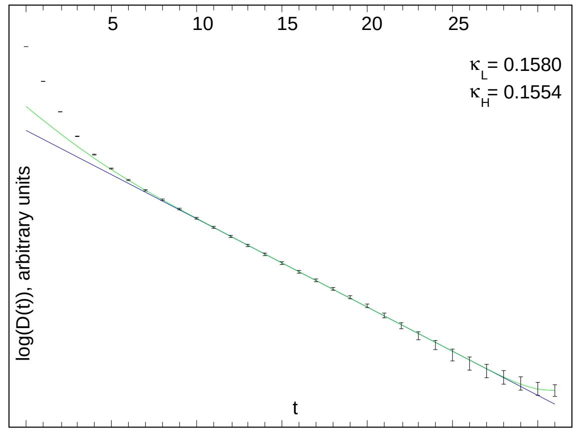

Figure 1. Lambda Propagator

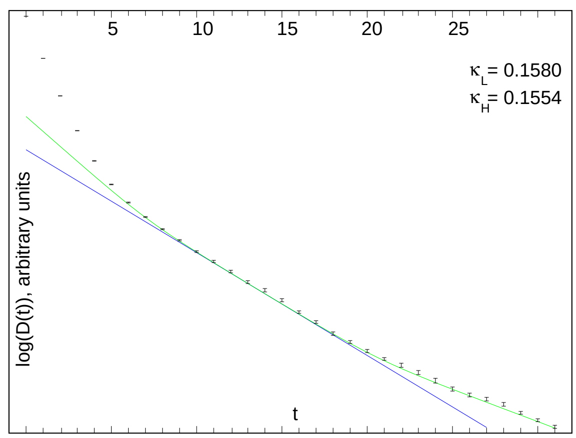

Figure 2. Dibaryon Propagator

finite size effects and quenching errors. On both lattice sizes we obtained comparable results, and, as expected [5], m_ρ is about 10% lighter than the experimental value.

In the strange sector, having set the strange quark mass by m_Λ, the lattice results [and corresponding experimental values] are: $m_\phi = 990(6)\,\mathrm{MeV}$ [$1020\,\mathrm{MeV}$], $m_K = 600(5)\,\mathrm{MeV}$ [$(m_{K^+} + m_{K^0})/2 = 496\,\mathrm{MeV}$], $m_{K^*} = 840(20)\,\mathrm{MeV}$ [$894\,\mathrm{MeV}$] and $m_\Xi = 1360(34)\,\mathrm{MeV}$ [$(m_{\Xi^0} + m_{\Xi^-})/2 = 1318\,\mathrm{MeV}$]. The errors are comparable with other quenched calculations.

Table 1
Masses of the H and Λ in $24^3 \times 32$ lattice.

		ma^{-1}	
κ_H	κ_L	H	Λ
0.1520	0.1568	2.091(09)	1.0143(06)
	0.1580	1.988(10)	0.9602(08)
	0.1592	1.858(14)	0.9053(17)
	0.1601	1.754(20)	0.8586(24)
0.1539	0.1568	2.021(08)	0.9790(07)
	0.1580	1.930(10)	0.9249(10)
	0.1592	1.799(15)	0.8681(20)
	0.1601	1.680(22)	0.8209(23)
0.1554	0.1568	1.968(07)	0.9501(08)
	0.1580	1.878(11)	0.8950(12)
	0.1592	1.753(17)	0.8371(23)
	0.1601	1.642(26)	0.7876(23)
0.1568	0.1568	1.916(07)	0.9221(10)
	0.1580	1.823(12)	0.8647(15)
	0.1592	1.705(22)	0.8060(26)
	0.1601	1.605(33)	0.7559(25)

6. *H* mass

For the lattice sizes we are able to treat, m_H has strong finite lattice size effects.

On the $16^3 \times 32$ lattice, the extrapolation to the physical m_u and m_s gives $m_H = 1950(60)$ MeV, just above the deuteron threshold. However, the 2 fm lattice appears too small to rule out considerable finite size effects.

Results from the larger lattice, $24^3 \times 32$ at the same value of β yield an unbound H at 2340(20) MeV. The dibaryon remain heavier than two Λ's at all combinations of the hopping parameters, as shown in table 1.

The sign and magnitude of the finite volume dependence indicate that interactions with periodic images are attractive, and significant at a separation of 2 fm. Since the attraction decreases with increasing volume and the H is already unbound by the order of 110 MeV on $24^3 \times 32$ lattice, we believe the infinite volume extrapolation of quenched QCD is even more unbound.

7. Comparison with Previous Work

There were two previous attempts to calculate m_H on the lattice a decade ago. Mackenzie and Tacker [3] did not see a bound dibaryon on a rather small lattice $6 \times 6 \times 12 \times 18$. Three years later Iwasaki, Yoshié and Tsuboi [4] used a larger lattice ($16^4 \times 48$) and obtained a very strongly bound H, between 1450(250) MeV and 1710(140) MeV, depending on the extrapolation. Since they used an unconventional gauge action and quark masses considerably heavier than those used in our calculation, direct comparison of numerical results is difficult. However, their result is consistent with the fact that interactions with images tend to overbind.

8. Conclusion and Future Plans

To strengthen our preliminary conclusion that the H is unbound in quenched QCD, we intend to recalculate the small lattice propagators at the same hopping parameters at the large lattice, so that more direct study of the finite size effects can be made. We also will calculate the two-point functions for dibaryons with other quantum numbers. In addition, we will extrapolate hadron masses in m_π and m_K rather than in the quark masses.

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REFERENCES

1. R.L. Jaffe, Phys. Rev. Lett. **38** (1977) 195.
2. K.F. Liu and C.W. Wong, Phys. Lett. **113B** (1982) 1; A.P. Balachadran, F. Lizzi and V.G. Roggers, Nucl. Phys. **B256** (1985) 525.
3. P.B. Mackenzie and H.B. Tacker; Phys. Rev. Lett. **55** (1985) 2539.
4. Y. Iwasaki, T. Yoshié and Y. Tsuboi, Phys. Rev. Lett. **60** (1988) 1371.
5. S. Aoki *at al.*, Nucl. Phys. **B62** (1998) 161.